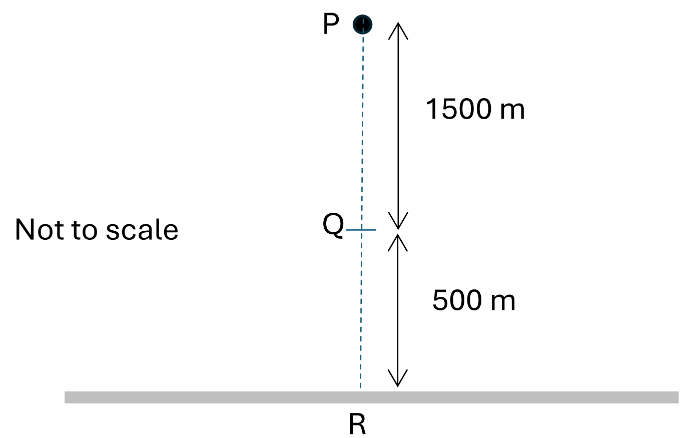
Teacher notes
Topic A

A simple but important problem

A ball of mass 60 kg is dropped from rest at P. A drag force *F* opposes the motion. By the time the ball gets to Q it has acquired a terminal speed of 100 m s^{-1}. The ball hits the ground at R.

The distance PQ is 1500 m and the distance QR is 500 m.



What is the work done by the drag force

(a) from P to Q
(b) from Q to R

Answer

We can answer this question using either $W_{total} = \Delta E_K$ or $W_{ext} = \Delta E_{total}$. We measure heights from the ground up so we have zero gravitational PE on the ground.

Using $W_{total} = \Delta E_K$:

(a) The total work done is W_F by the drag force plus the work done by the weight, mgh_{PQ}:

$W_F + mgh_{PQ} = W_F + 60 \times 10 \times 1500 = W_F + 9.0 \times 10^5$

The change in KE is $\frac{1}{2} \times 60 \times 100^2 - 0 = 3.0 \times 10^5$ J.

Hence

$W_F + 9 \times 10^5 = 3 \times 10^5$ and so $W_F = -6.0 \times 10^5$ J.

(b) From Q to R the drag force equals the weight, 600 N, and so the work done is just $W_F = -600 \times 500 = -3.0 \times 10^5$ J. This is the easiest way to answer this.

Using $W_{total} = \Delta E_K$ we would find

$W_F + mgh_{QR} = W_F + 60 \times 10 \times 500 = W_F + 3.0 \times 10^5$

The change in KE from Q to R is zero and so $W_F + 3.0 \times 10^5 = 0 \Rightarrow W_F = -3.0 \times 10^5$ J.

Using $W_{ext} = \Delta E_{total}$:

(a) From P to Q,

$$\Delta E_{total} = \frac{1}{2} mv^2 + mgh_{QR} - (mgh_{PR} + 0)$$
$$= \frac{1}{2} \times 60 \times 100^2 + 60 \times 10 \times 500 - 60 \times 10 \times 2000$$
$$= -6.0 \times 10^5$$

The only external force is the drag force and so $W_F = -6.0 \times 10^5$ J.

(Remember that by external forces we mean non-conservative forces; so, the weight is not part of the external forces. The weight is a conservative force indicated by the fact that we include gravitational potential energy as part of the total mechanical energy of the system.)

(b) Using this method for this part gives

$$\Delta E_{\text{total}} = (\frac{1}{2}mv^2 + 0) - (\frac{1}{2}mv^2 + mgh_{\text{QR}})$$
$$= -60 \times 10 \times 500$$
$$= -3.0 \times 10^5 \text{ J}$$

The only external force is the drag force and so $W_F = -3.0 \times 10^5$ J.